On the Revealed Preference Theory of Two-sided Matching Models with One-sided Preferences*

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Abstract

This paper studies the testable implications of the stable matching theory with one-sided preferences. We establish the connection between the standard revealed preference analysis and the well-known lattice structure of stable matchings in the literature. We also revisit the Rural Hospital Theorem from the perspective of revealed preference.

Keywords: stable matching; revealed preference; lattice structure; Rural Hospital Theorem

1 Introduction

Revealed preference theory is based on the idea that the preferences of the agents can be revealed from their choice behavior.[1] That is, the choice of one option over another conveys preference. To draw inferences from the behavior, one need not only to observe the chosen option, but also what options are available. The latter is not obvious in matching markets as in, say, consumer theory.[2] For example, in a two-sided matching model, if Catherine is matched with Jules and not with Jim, it may be because she likes Jules best, but it may also be because she likes Jim best but Jim is matched to someone he prefers over Catherine.

In this paper, we study the testable implications of stability in two-sided matching models such as school choice, marriage matching, etc. We present our results using the language of school choice where students are matched with schools. It is assumed that the schools' priority orderings are known. A data set of matchings, in which each matching assigns students to schools, is said to be *rationalizable* if there exists a profile of student preferences such that every matching in the data set is stable given the students' preferences and the schools' priorities. Given a data set, we first identify the available options for

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[1]See Chambers and Echenique (2016) for a comprehensive survey.

[2]See Afriat (1967), where the latter requirement is implicit and satisfied as a by-product of the observation itself. See the survey Roth and Sotomayor (1992) for two-sided matching theories.

each student explicitly and characterize the rationalizability of a data set by the standard acyclicity condition (Theorem 1). Secondly, we establish the connection between the standard revealed preference analysis and the lattice structure of stable matchings, which says that the set of stable matchings forms a distributive lattice.[3] More precisely, the rationalizability of a data set can be tested by an algorithm based on the join and meet operators in the lattice theory (Theorem 2). Thirdly, we conclude the paper by revisiting the Rural Hospital Theorem from the perspective of revealed preference (Proposition 3).

We are not the first to combine the revealed preference literature and the matching literature. Echenique (2008) studies testable implications of matching theory when the preferences of agents on both sides of the market are unknown. On the one hand, our setting is a special case of his. On the other hand, however, the novel observation we make here (Theorem 2) is only available when the preferences of agents on one side of the market are known (in our language, school priorities are known). Echenique et al. (2013) investigates the revealed preference theory of aggregate matchings, which differs from a collection of individual-level matchings studied in Echenique (2008) and the current paper.

Haeringer and Iehlé (2014) also studies matching with one-sided preferences. However, they focus on how one can identify impossible matches in stable matchings, by considering only the preferences on one side of the market. Indeed, impossible matches can exclude some matching from being stable. Given a collection of the survived matchings, our paper studies whether the matchings in the collection are stable under some particular preference profile.

2 Model

2.1 School Choice Model

We consider the following setup of the school choice problem.[4] There is a finite set I of students to be matched with a finite set S of schools. Denote a generic student by i and a generic school by s. For simplicity, we assume one-to-one matching, i.e., each school can admit at most one student.

Each student $i \in I$ has a strict preference relation $\succ_i$ over $S \cup \{i\}$, where i stands for the outside option of remaining unmatched. We denote the relation that student i prefers school s to another school s' by $s \succ_i s'$. Similarly, the relation $s \succ_i i$ means that i prefers to be matched with school s rather than remaining unmatched. Each student i's preference $\succ_i$ over $S \cup \{i\}$ is her private information.

Each school $s \in S$ is endowed with a strict priority ordering over $I \cup \{s\}$, where s stands for the situation of being unmatched. We denote the relation that student i has higher

[3] See Blair (1984), Knuth (1976), and the comprehensive survey Roth and Sotomayor (1992) for the structural results of the set of stable matchings.

[4] See for example, Abdulkadiroglu and Sönmez (2003) for more discussions.

priority at school s than student i' by $i \succ_s i'$. Schools' priority orderings, $\{\succ_s\}_{s\in S}$, are publicly known.

A *school choice market* is denoted by $\Gamma = (I, S, \{\succ_s\}_{s\in S}, \{\succ_i\}_{i\in I})$. For notation convenience, we denote by $\{\succcurlyeq_v\}_{v\in I\cup S}$ the weak preference relations corresponding to $\{\succ_v\}_{v\in I\cup S}$.

2.2 Stable Matchings

A *matching* is a mapping $\mu : I \cup S \to I \cup S$ such that each student is either unmatched or assigned to a school that admits her, i.e.,

1. $\mu(i) = i$ if $\mu(i) \notin S$;
2. $\mu(i) = s$ if and only if $i = \mu(s)$ for all $s \in S$ and all $i \in I$.

Given (I, S), the set of all possible matchings is denoted by M. A matching μ is said to be *stable* if it is *individually rational*, i.e., no agent's match is worse than being unmatched and *not blocked*, i.e., there is no justified envy; formally, μ is said to be stable if

1. $\mu(i) \succcurlyeq_i i$ for all $i \in I$;
2. $s \succ_i \mu(i)$ implies $\mu(s) \succ_s i$ for all $s \in S$ and $i \in I$.

The set of stable matchings for market Γ is denoted by $\Sigma(\Gamma)$, or Σ for simplicity. For an arbitrary matching μ, the pair (i, s) is a *blocking pair* for μ if $s \succ_i \mu(i)$ and $i \succ_s \mu(s)$.

The preference of a generic agent $v \in I \cup S$ over matchings in M is induced by the preference (priority) $\succ_v$ over individual agents, i.e., for any two matchings $\mu, \mu' \in M$, (abusing the notions $\succ$ and $\succcurlyeq$) $\mu \succ_v \mu'$ (respectively $\mu \succcurlyeq_v \mu'$) if and only if $\mu(v) \succ_v \mu'(v)$ (respectively $\mu(v) \succcurlyeq_v \mu'(v)$).

2.3 Rationalizability of a Matching Data Set

A *data set* D is a collection of matchings: $D = \{\mu_k\}_{k\in K}$, where K is a finite index set. The interpretation of a data set D is that it describes K observations of the matching outcomes for a school choice market.

We say that a data set D is *rationalizable* if there exists a student preference profile $\{\succ_i\}_{i\in I}$ such that $D \subseteq \Sigma(\Gamma)$, where $\Gamma = (I, S, \{\succ_s\}_{s\in S}, \{\succ_i\}_{i\in I})$. Since $(I, S, \{\succ_s\}_{s\in S})$ are fixed, we simplify the notation to write $\Sigma(\Gamma)$ as $\Sigma(\{\succ_i\}_{i\in I})$.

2.4 Technical Definitions

A *directed graph* is a pair $G = (V, L)$, where V is a set of *vertex* and L is a subset of $V \times V$ that is called *edges*. A *path* in G is a sequence $p = \langle v_0, \ldots, v_N\rangle$ such that $(v_n, v_{n+1}) \in L$ for all $n \in \{0, \ldots, N-1\}$. We denote by $v \in p$ that v is a vertex in p. A *cycle* in G is a path $c = \langle v_0, \ldots, v_N\rangle$ with $v_0 = v_N$, where we say the *length* of it is N. For the rest of the paper, V refers to the set of schools S or the set of all agents $I \cup S$, which we do not distinguish when there is no ambiguity.

3 Results

3.1 Revealing Students' Preferences

In this subsection, we identify the available options for each student explicitly and characterize the rationalizability of a data set by the standard acyclicity condition.

Given a matching $\mu_k \in D$, a school $s \in S$ is available for a student $i \in I$ if s prefers i to its own student $\mu_k(s)$, i.e., $i \succ_s \mu_k(s)$. More formally, for each $k \in K$ and each school $s \in S$, the upper contour set (UCS) of school s at the matching μ_k is given by

$$U(s, \mu_k) := \{i \in I : i \succ_s \mu_k(s)\}.$$

Therefore, the available schools for i under μ_k is

$$A(i, \mu_k) := \{s \in S : i \in U(s, \mu_k)\}.$$

If D is rationalizable, then $\mu_k(s)$ is a stable matching. Thus $i \succ_s \mu_k(s)$ must reveals that i prefers her own school to s. Following this idea, the revealed preference relation for student i is collected in the set

$$R(i) := \bigcup_{k \in K} \{(\mu(i), s) : s \in A(i, \mu_k)\}.$$

By construction, $(S \cup \{i\}, R(i))$ is a directed graph for every $i \in I$. The theorem below characterizes the rationalizability of a data set.

Theorem 1. *D is rationalizable if and only if $(S \cup \{i\}, R(i))$ admits no cycle for all $i \in I$.*

The proof of Theorem 1 is standard and thus omitted. Nevertheless, we illustrate it by the following example.

Example 1. *Consider the matchings between students in $I = \{i_1, i_2, i_3\}$ and schools in $S = \{s_1, s_2, s_3\}$. The priorities for schools are given as follows.*

$$s_1 : i_2 \succ_{s_1} i_3 \succ_{s_1} i_1 \succ_{s_1} s_1$$
$$s_2 : i_3 \succ_{s_2} i_1 \succ_{s_2} i_2 \succ_{s_2} s_2$$
$$s_3 : i_1 \succ_{s_3} i_2 \succ_{s_3} i_3 \succ_{s_3} s_3$$

The data set contains two observations, i.e., $D = \{\mu, \mu'\}$, where[5]

$$\mu = \begin{array}{ccc} i_1 & i_3 & i_2 \\ | & | & | \\ s_1 & s_2 & s_3 \end{array} \quad \textit{and} \quad \mu' = \begin{array}{ccc} i_3 & i_2 & i_1 \\ | & | & | \\ s_1 & s_2 & s_3 \end{array}.$$

[5]We write a matching in such a way for convenience, where, for example, $\mu(i_1) = s_1$, $\mu(i_3) = s_2$, and $\mu(i_2) = s_3$.

Now we conduct the test shown in Theorem 1. The results are summarized in Table 1. Since $c = \langle s_1, s_2, s_1\rangle$ is a cycle in the graph $(S \cup \{i_3\}, R(i_3))$, we know D is not rationalizable.

Table 1: Revealed Preferences of the Students in Example 1.

Matchings	Schools	UCSs	Revealed Preferences		
			$R(i_1)$	$R(i_2)$	$R(i_3)$
μ	s_1	$\{i_2, i_3\}$		(s_3, s_1)	(s_2, s_1)
	s_2	$\emptyset$			
	s_3	$\{i_1\}$	(s_1, s_3)		
μ'	s_1	$\{i_2\}$		(s_2, s_1)	
	s_2	$\{i_1, i_3\}$	(s_3, s_2)		(s_1, s_2)
	s_3	$\emptyset$			

3.2 Structure of the Observed Matchings

In this subsection, we establish the connection between the standard revealed preference analysis in subsection 3.1 and the lattice structure of stable matchings.

We first introduce some notations for the lattice theory. For any two matchings μ and μ', we can define their *join* (with respect to $\{\succ_s\}_{s\in S}$), where each school s is assigned to its preferred student between $\mu(s)$ and $\mu'(s)$. Formally, the join of μ and μ' is denoted by $\lambda = \mu \vee \mu'$, where for each $s \in S$, $\lambda(s) = \mu(s)$ if $\mu(s) \succ_s \mu'(s)$ and $\lambda(s) = \mu'(s)$ otherwise. In a precisely similar way we can define the *meet* of μ and μ' (with respect to$\{\succ_s\}_{s\in S}$), which is denoted by $\nu = \mu \wedge \mu'$. Formally, for all $s \in S$, $\nu(s) = \mu(s)$ if $\mu'(s) \succ_s \mu(s)$ and $\nu(s) = \mu'(s)$ otherwise.

We impose the *partial order* $\leq$ on the set M, where $\mu \leq \mu'$ if and only if $\mu'(s) \succsim_s \mu(s)$ for all $s \in S$. The partially ordered set $(M, \leq)$ is called a *lattice* if each two-element subset $\{\mu, \mu'\} \subseteq M$ has a join and a meet in M. As is well known in the literature, the set of stable matchings forms a lattice (see Knuth (1976), who attributes the result to John Conway). That is, if μ and μ' are both stable matchings, then both $\lambda = \mu \vee \mu'$ and $\nu = \mu \wedge \mu'$ are matchings and they are stable. The lattice structure of stable matchings can be used to test the rationalizability of a data set, which we illustrate by the following example.

Example 2. *(Example 1 revisited.) The data set and the school priorities are the same as in Example 1. We calculate the join and meet of the two matchings μ and μ' as follows.*

$$\lambda = \mu \vee \mu' = \begin{matrix} i_3 & i_3 & i_1 \\ s_1 & s_2 & s_3 \end{matrix} \quad \textit{and} \quad \nu = \mu \wedge \mu' = \begin{matrix} i_1 & i_2 & i_2 \\ s_1 & s_2 & s_3, \end{matrix}$$

Suppose D is rationalizable. Then both λ and ν are stable matchings. Now that neither of them is even a matching, we know that D must not be rationalizable.

Generally, if a data set D is rationalizable, then the join and meet of any two matchings in D must be well-defined matchings. We proceed to show that the converse is almost true. Before formalizing our statement, we introduce some notations. Define $D_0 = D$. For each integer $l \geq 0$, define two sets of mappings D_{l+1} and D_{-l-1} from $I \cup S$ to $I \cup S$ such that

$$\begin{aligned} D_{l+1} :=& \{\mu \vee \mu' : \mu, \mu' \in D_l\} \text{ and} \\ D_{-l-1} :=& \{\mu \wedge \mu' : \mu, \mu' \in D_{-l}\}. \end{aligned}$$

Since we allow for $\mu = \mu'$, $D_{l+1} \supseteq D_l$ and $D_{-l-1} \supseteq D_{-l}$ for all integer l. Let $\bar{l}$ be the smallest integer such that $D_{\bar{l}+1} = D_{\bar{l}}$, and similarly $\underline{l}$ the largest integer such that $D_{-\underline{l}-1} = D_{-\underline{l}}$. Both $\bar{l}$ and $\underline{l}$ exist and are finite because we have finite agents and thus finitely many mappings from $I \cup S$ to $I \cup S$. Let DL be all the mappings derived by applying the join and meet operators on D, i.e., $DL := \bigcup_{l=-\underline{l}}^{\bar{l}} D_l$. Let $N(\mu)$ be the set of unmatched agents under the matching $\mu \in D$, i.e.,

$$N(\mu) := \{v \in I \cup S : \mu(v) = v\} \text{ for all } \mu \in D.$$

The following theorem shows that a data set is rationalizable if and only if (i) the set of unmatched agents is the same for all data points and (ii) every product of the two operators are well-defined matchings.[6]

Theorem 2. *D is rationalizable if and only if*

1. *$N(\mu) = N(\mu')$ for any two matchings $\mu, \mu' \in D$; and*
2. *$DL \subseteq M$.*

The insufficiency of the second condition is illustrated by the following example.

Example 3. *Consider the matchings between one student i and one school s, i.e., $I = \{i\}$ and $S = \{s\}$. The priority order of s is trivial, i.e., $i \succ_s s$. The data set contains two observations, i.e., $D = \{\mu, \mu'\}$, where $\mu(i) = s$, $\mu(s) = i$, $\mu'(i) = i$ and $\mu'(s) = s$. Obviously, $\mu' \leq \mu$. Therefore, $DL = D = \{\mu, \mu'\} \subseteq M$. However, D is not rationalizable; otherwise the stability of μ' implies that $i \succ_i s$, which contradicts the stability of μ.*

Proof of Theorem 2. Suppose D is rationalizable. Then there exists a preference profile for students, $\{\succ_i\}_{i \in I}$ such that $D \subseteq \Sigma(\{\succ_i\}_{i \in I})$. The first condition is true by, for example, Theorem 2.22 of Roth and Sotomayor (1992). For the second, we use the lattice result of Knuth (1976), which says that $D_1 \subseteq \Sigma(\{\succ_i\}_{i \in I})$ and $D_{-1} \subseteq \Sigma(\{\succ_i\}_{i \in I})$. By induction in l, we know that $D_l \subseteq \Sigma(\{\succ_i\}_{i \in I})$ for all $l = \underline{l}, \ldots, \bar{l}$. Therefore, $DL \subseteq \Sigma(\{\succ_i\}_{i \in I}) \subseteq M$.

Conversely, suppose the two conditions hold. We construct the student preference profile to rationalize DL, and thus D, as follows. For any two matchings $\mu', \mu \in DL$ such that $\mu \leq \mu'$ and every student $i \in I$, if $\mu'(i) \neq \mu(i)$, then $\mu(i) \succ_i \mu'(i)$. Since $\leq$ on M is a

[6] Condition (i) is known as the *Rural Hospital Theorem* or *Lone Wolf Theorem*. See Section 4.

partial order, we know that $\succ_i$ is a strict partial order.[7] Therefore, $\succ_i$ can be extended to a strict preference order on $S \cup \{i\}$.[8] We take a particular extension with the following properties. That is, for every $i \in I$, every school that was never matched to i is in the tail of i's preference list and the order is arbitrary, i.e., (abusing the notation $\succ_i$)

$$s \succ_i s', \text{for all } s \in \{\mu(i) : \mu \in DL\} \text{ and all } s' \in S \backslash \{\mu(i) : \mu \in DL\}.$$

Take an arbitrary $\mu \in DL$. We proceed to argue that $\mu \in \Sigma(\{\succ_i\}_{i \in I})$. If μ is not individually rational under $\{\succ_i\}_{i \in I}$, then there exists a student i such that $i \succ_i \mu(i)$. By the construction of $\succ_i$, $i \succ_i \mu(i)$ only if $\mu'(i) = i$ and $\mu'(i) \neq \mu(i)$ for some $\mu' \in DL$. Since $\mu'(i) \neq \mu(i)$, we know that $\mu(i) \neq i$. Therefore, $i \notin N(\mu)$ but $i \in N(\mu')$, a contradiction to condition 1. Therefore, μ is individually rational.

Suppose to the contrary that μ is blocked, i.e., there exists a pair (i, s) such that $s \succ_i \mu(i)$ and $i \succ_s \mu(s)$. By the construction of students' preference profile, $s \succ_i \mu(i)$ only if there exists $\mu' \in DL$ such that $\mu'(i) = s$ and $\mu' \leq \mu$, where the latter implies that $\mu(s) \succ_s \mu'(s) = i$, a contradiction. Therefore, every matching in DL, and thus every matching in $D \subseteq DL$, is stable. $\square$

The preference construction above is consistent with the well-known results that the agents on one side of the market have a common interest regarding the set of stable matchings and that agents on the opposite side of the market have opposite common interest in this regard (see, for example, (Roth and Sotomayor, 1992, Theorem 2.13)). Theorem 2, as Theorem 1, provides an algorithm to test whether a data set is rationalizable, i.e. simply computing the joins and meets to see if all results are proper matchings.

4 A Necessary Condition of Rationalizability: Revisiting the Rural Hospital Theorem[9]

It is well known in the literature that with strict preferences, the set of agents who are unmatched is the same for all stable matchings, which is known as the Rural Hospital Theorem or the Lone Wolf Theorem.[10] This property serves as a necessary condition for rationalizability as in Theorem 2 (condition 1) and in the more general setting where the schools' priorities are unknown just as the students' preferences. To be precise, Theorem 3 below holds in such a general setting, since its proof does not depend on the known school priorities.

We revisit the Rural Hospital Theorem/Lone Wolf Theorem from the revealed-preference

[7] A *strict partial order* is a binary relation that is irreflexive, transitive and strictly antisymmetric. See, for example, Flaška et al. (2007).

[8] See, for example, Theorem 1.5 of Chambers and Echenique (2016).

[9] In some context, it is called the Lone Wolf Theorem. See, for example, Ciupan et al. (2016).

[10] See, for example, Theorem 2.22 of Roth and Sotomayor (1992).

point of view and present an elementary proof in this section. The standard proof of the theorem relies on the decomposition lemma (see (Roth and Sotomayor, 1992, pp. 41-43)) or special cases of the lemma (see (Roth, 1984, Theorem 9)). In an independent paper, Ciupan et al. (2016) provides an elementary proof of the theorem by considering the market equilibration process of Blum and Rothblum (2002). Particularly, the proof relies on the result that the market equilibration process always produces a stable matching under which the last agent to enter receives his or her best stable matching partner. We rewrite the Rural Hospital Theorem/Lone Wolf Theorem in the following theorem, using the revealed-preference language.

Theorem 3. *If D is rationalizable, then the set of students and schools who are unmatched is the same for all matchings in D.*

Proof. Suppose to the contrary that there exist two matchings $\mu, \mu' \in D$ such that $\mu(i) = i$ while $\mu'(i) \neq i$ for some student $i \in I$ (the case where a school is matched under one matching while unmatched under another is symmetric). Consider the path constructed by alternatively applying μ' and μ to i until some agent is reached for a second time (shown in Figure 1), i.e., $p = \langle i, \mu'(i), \mu(\mu'(i)), \mu'(\mu(\mu'(i))), \ldots, (\mu \circ \mu')^k \circ i \rangle$ or $p = \langle i, \mu'(i), \mu(\mu'(i)), \mu'(\mu(\mu'(i))), \ldots, \mu' \circ (\mu \circ \mu')^k(i) \rangle$ for some integer k, where the construction stops when encountering an existing agent (this is guaranteed in Claim 1 below). Obviously, the path p involves finitely many agents because $I \cup S$ is finite, which implies that k is finite.

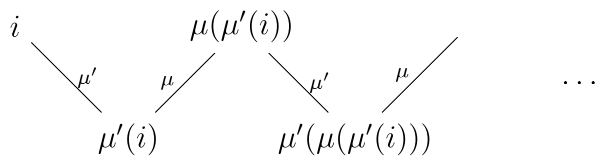

Figure 1: An sequence by alternatively applying μ and μ' to i.

Claim 1. *1. The path p admits a cycle, i.e. there exists an l with $0 \leq l < k$ such that*

$$\textit{either } (\mu \circ \mu')^k \circ i = (\mu \circ \mu')^l \circ i \tag{1}$$
$$\textit{or } (\mu' \circ \mu)^k \circ \mu'(i) = (\mu' \circ \mu)^l \circ \mu'(i). \tag{2}$$

2. Suppose D is rationalizable by $\{\succ_v\}_{v \in I \cup S}$. Then for every l such that $0 \leq l \leq k-1$,

$$(\mu \circ \mu')^{l+1} \circ i \succ_{\mu'((\mu\circ\mu')^l \circ i)} (\mu \circ \mu')^l \circ i \tag{3}$$
$$\textit{and } (\mu' \circ \mu)^{l+1} \circ \mu'(i) \succ_{\mu((\mu'\circ\mu)^l \circ \mu'(i))} (\mu' \circ \mu)^l \circ \mu'(i). \tag{4}$$

We consider only the case of (1) stated in Claim 1, and the argument for the other case is symmetric. Suppose there exists some preference-priority profile $\{\succ_v\}_{v \in I \cup S}$ such that μ

and μ' are both stable, and $l \geq 1$ in (1). Since $\mu'((\mu \circ \mu')^{k-1} \circ i)$ is the partner of $(\mu \circ \mu')^k \circ i$ under μ, and $\mu'((\mu \circ \mu')^{l-1} \circ i)$ is the partner of $(\mu \circ \mu')^l \circ i$ under μ, we know from (1) that

$$\mu'((\mu \circ \mu')^{k-1} \circ i) = \mu'((\mu \circ \mu')^{l-1} \circ i).$$

This implies that p should have stopped with $(\mu \circ \mu')^k \circ i$ by the stopping criterion of p, a contradiction. When $l = 0$, i has a partner $\mu'((\mu \circ \mu')^{k-1} \circ i)$ under μ, which is a school by (4), a contradiction. Therefore, it is impossible to find a preference profile such that both μ and μ' are stable. $\square$

Proof of Claim 1. We show (3)-(4) and then (1)-(2). First, $\mu'(i) \succ_i \mu(i) = i$; otherwise μ' is unstable (not individually rational for i). If $\mu(\mu'(i)) = \mu'(i)$, then μ is unstable because it is blocked by $(i, \mu'(i))$. Thus, $\mu(\mu'(i)) \neq \mu'(i)$. Second, $\mu(\mu'(i)) \succ_{\mu'(i)} i$; otherwise μ is unstable because it is blocked by $(i, \mu'(i))$. Proceed by induction, we have (3) and (4). Since the market has finitely many agents, if (1) and (2) both fail then the last element of p must be matched with herself (itself), which contradicts either (3) or (4). Hence, at least one of (1) and (2) holds. $\square$

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